

## CHAPTER 8 – 3D SHAPES REVIEW

Show work for EVERY QUESTION.

$$r = 1.5$$

1) A closed cylindrical can has a diameter of 3 ft and a height of 1.8 ft.

- Find:
- The volume of the can correct to the nearest tenth of a cubic foot.
  - The lateral area of the can in terms of  $\pi$ .
  - The surface area of the can correct to the nearest tenth of a square foot.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi (1.5)^2 (1.8) \\ &= 12.7 \text{ ft}^3 \end{aligned}$$

$$\begin{aligned} LA &= 2\pi r h \\ &= 2\pi (1.5)(1.8) \\ &= 5.4\pi \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} BA &= \pi r^2 \\ &= \pi (1.5)^2 \\ &= 2.25\pi \text{ (2 Bases)} \\ &= 4.5\pi \end{aligned}$$

$$\begin{aligned} SA &= 5.4\pi + 4.5\pi \\ &= 31.1 \text{ ft}^2 \end{aligned}$$

2) The amount of light produced by a cylindrical-shaped fluorescent light bulb depends on its lateral area. A certain cylindrical-shaped fluorescent light bulb is 36 inches in length, has a 1-inch diameter, and is manufactured to produce 0.283 watts of light per square inch. What is the best estimate for the total amount of light that it is able to produce?

- (1) 32 watts     (2) 34 watts     (3) 48 watts     (4) 64 watts

$$\begin{aligned} LA &= 2\pi r h \\ &= 2\pi (.5) 36 \\ &= 36\pi \cdot .283 \\ &= 32 \end{aligned}$$

3) The lateral area of a right cylinder whose height is two times the length of the radius is  $100\pi$  square inches. What is the number of inches in the height of the cylinder?

- (1) 5     (2) 10     (3) 15     (4) 20

$$2x = 10$$

$$\begin{aligned} LA &= 2\pi r h \\ 100\pi &= 2\pi x \cdot 2x \\ 100\pi &= 4\pi x^2 \\ x &= 5 \end{aligned}$$

4) A right circular cylinder has a lateral area of  $306\pi \text{ ft}^2$ . If the height of the cylinder is 17 ft, express in terms of  $\pi$  the number of cubic feet in the volume of the cylinder.

$$LA = 2\pi r h$$

$$\frac{306\pi}{34\pi} = \frac{2\pi r \cdot 17}{34\pi}$$


$$9 = r$$

$$V = \pi r^2 h$$

$$= \pi (9)^2 \cdot 17$$

$$\boxed{V = 1377\pi \text{ ft}^3}$$

5) For any regular pyramid with height  $h$  and slant height  $l$ , which statement is always true?


- (1)  $h > l$     (2)  $h < l$     (3)  $h = l$     (4)  $l >$  lateral edge
- 

6) If a regular square pyramid with side length  $s$  and a right cone with radius  $r$  have congruent altitudes and equal volumes, then

(1)  $s = \sqrt{\pi r}$     (2)  $s = \frac{\sqrt{r}}{\pi}$     (3)  $s = \pi\sqrt{r}$     (4)  $s = r\sqrt{\pi}$

$$V_p = \frac{1}{3} B h = \frac{1}{3} s^2 h$$

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 h$$

$$s^2 = \pi r^2 \implies s = r\sqrt{\pi}$$


7) For any right cone, which statement is always true?

- (1) lateral area > area of base  
 (2) area of base > lateral area  
 (3) area of base = lateral area  
 (4) surface area = 2(area of base)

$$LA_{\text{cone}} = \pi r l = \pi(3)(5) = 15\pi$$

$$A_o = \pi r^2 = \pi(3)^2 = 9\pi$$

8) In a square pyramid, the slant height is 15 cm, and the length of a side of the base is 24 cm. Find the lateral area and volume of the prism.

$$LA = \frac{1}{2} P h$$

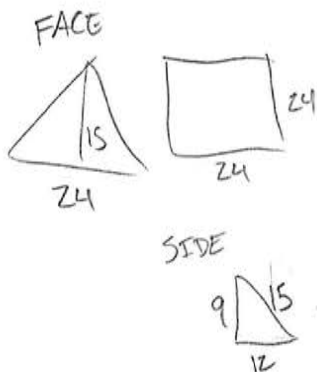
$$= \frac{1}{2} 96 \cdot 15$$

$$= 720 \text{ cm}^2$$

$$V = \frac{1}{3} B h$$

$$= \frac{1}{3} 576 \cdot 9$$

$$= 1728 \text{ cm}^3$$



9) If the diameter of a sphere is tripled, the volume of the sphere is multiplied by

(1) 4

(2) 9

(3) 16

(4) 27

$3^3$

10) What is the best approximation of the number of square inches in the surface area of a spherical ball that fits tightly in a cube-shaped box with edges 8 inches in length?

(1) 67

(2) 201

(3) 268

(4) 804

$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4\pi(4)^2 \\ &= \end{aligned}$$

11) If the volume of a sphere is  $7,776\pi$  cubic centimeters, what is the number of square centimeters in the surface area of the sphere, rounded to the nearest square centimeter?

$$\frac{7776\pi}{\left(\frac{4}{3}\pi\right)} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi}$$

$$\begin{aligned} 5832 &= r^3 \\ r &= 18 \end{aligned}$$

$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4\pi(18)^2 \\ &= 4072 \text{ cm}^2 \end{aligned}$$

12) Lauren has a softball whose circumference measures 13 inches. She wants to box it for a gift for Nicole but can only find cube-shaped boxes of 3 inches, 4 inches, 5 inches, or 6 inches. Lauren is cheap and wants to buy the smallest possible box to wrap the softball.

a. What is the smallest box that the ball will fit into with the top on? 5 in

b. Lauren has a square sheet of wrapping paper that measures 1 ft by 1 ft. Assuming no waste, does Lauren have enough wrapping paper to completely cover the box? Explain your answer.

$$\begin{aligned} C &= \pi d \\ 13 &= \pi d \\ d &= 4.138 \end{aligned}$$

$$\begin{aligned} BA &= 5 \cdot 5 \\ &= 25 \cdot 6 \text{ sides} \\ SA &= 150 \end{aligned}$$

$$\begin{aligned} &1 \text{ by } 1 \\ &12 \text{ by } 12 = 144 \\ &\text{No } SA > 144 \text{ in}^2 \end{aligned}$$

13) Two planes, A and B, intersect the same sphere at the same perpendicular distance from the center of the sphere. The area of the circle formed by the intersection of plane A is  $225\pi$ . What is the diameter of the circle formed by the intersection of plane B and the sphere?

$$\begin{aligned} A &= \pi r^2 \\ 225\pi &= \pi r^2 \\ 225 &= r^2 \\ r &= 15 \\ d &= 2r \\ &= 2 \cdot 15 \\ &= 30 \end{aligned}$$

14) The area of the base of a pyramid is 24 and its altitude is 10. Find the volume of the pyramid.

$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3} 24 \cdot 10$$

$$\boxed{V = 80}$$

15) A right circular cylinder and a right cone have equal bases and equal lateral areas. The altitude of the cylinder is 7. Find the slant height of the cone.

$$LA_{\text{cyl}} = 2\pi r h \qquad LA_{\text{cone}} = \pi r l$$

$$\frac{2\pi r(7)}{\pi r} = \frac{\pi r l}{\pi r}$$

$$\boxed{14 = l}$$

16) The volume of a sphere is  $512\pi$  cubic millimeters. What is the radius of this sphere to the nearest tenth of a millimeter?

$$V = \frac{4}{3} \pi r^3$$

$$512\pi = \frac{4}{3} \pi r^3$$

$$384 = r^3$$

$$\boxed{r = 7.3 \text{ mm}}$$

17) The altitude of a right circular cylinder is equal to the diameter of its base. The lateral area of the cylinder is  $144\pi$  square units. Find the area of its base.

$$LA = 2\pi r h$$

$$144\pi = 2\pi r(2r)$$

$$144\pi = 4\pi r^2 \qquad r = 6$$

$$36 = r^2$$

$$h = d$$

$$h = 2r$$

$$BA = \pi r^2$$

$$= \pi(6)^2$$

$$\boxed{BA = 36\pi \text{ sq units}}$$

18) A sealed cylindrical can holds three tennis balls, each with a diameter of 3.5 inches. If the can is designed to have the smallest possible volume, find the number of cubic inches of *unoccupied* space inside the can correct to the *nearest tenth* of a cubic inch.

$$r = 1.75$$

$$h = 3 \cdot 3.5 = 10.5$$

$$V_{\text{TB}} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \pi (1.75)^3$$

$$= 22.449298$$

$$= 67.347893 \text{ (3TB)}$$

$$V_{\text{can}} = \pi r^2 h$$

$$= \pi (1.75)^2 10.5$$

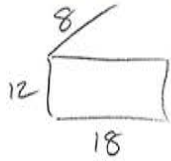
$$= 101.02184$$

$$- 67 \dots$$

$$\boxed{33.7 \text{ in}^3}$$

**All Things 2D and 3D figures:**

- 1) Determine the lateral area of a rectangular solid whose length is 18 in, width is 1 foot, and height is 8 in.



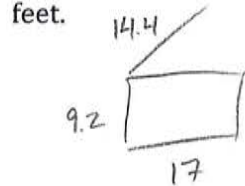
$$LA = Ph$$

$$= 60 \cdot 8$$

$$=$$

$$LA = 480 \text{ in}^2$$

- 2) Determine the surface area of a rectangular solid whose length is 17 in, width is 9.2 in, and height is 1.2 feet.



$$LA = Ph$$

$$= 52.4 \cdot 14.4$$

$$= 754.56$$

$$BA = 2lw$$

$$= 2(9.2)(17)$$

$$= 312.8$$

$$SA = 1067.36 \text{ in}^2$$

- 3) Determine the volume of a rectangular solid whose length is 9 cm, width is 120 mm, and height is 8.7 in.

$$V = Bh$$

$$= 108 \cdot 22.359$$

$$=$$

$$V = 2414.772 \text{ in}^3$$

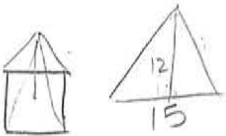
$$B = 9 \cdot 12$$

$$= 108$$

$$12 \text{ cm}$$

$$22.359 \text{ cm}$$

- 4) Determine the lateral and surface area of a pyramid with a square base if each edge of the square measure 15 cm (150 mm), the pyramid's height is 17 cm, and the slant height is 12 cm.



$$LA = \frac{1}{2} Ph$$

$$= \frac{1}{2} (60) 12$$

$$= 360 \text{ in}^2$$

$$BA = 15^2$$

$$= 225$$

$$LA = 360 \text{ in}^2$$

$$SA = 585 \text{ in}^2$$

- 5) Determine the volume of a pyramid with a square base if each edge of the base measures 8 in, the height of the pyramid is 9 in, and the slant height is 12 in.

$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3} (64) 9$$

$$V = 192 \text{ in}^3$$

$$BA = 8^2$$

$$= 64$$

- 6) Determine the lateral area of a cylinder with a diameter of 1 foot and a height of 18 cm, to the nearest tenth.

$$LA = 2\pi rh$$

$$= 2\pi (15.24) 18$$

$$LA = 1723.6 \text{ cm}^2$$

$$30.48 \text{ cm}$$

$$r = 15.24$$

- 7) Determine the volume of a cylindrical container whose height is 18 feet and whose diameter is 12 feet, to the nearest tenth.

$$V = \pi r^2 h$$

$$= \pi (6)^2 18$$

$$V = 2035.8 \text{ ft}^3$$

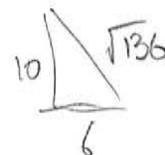
$$r = 6$$

- 8) Determine the lateral area of a cone whose radius is 6 cm, and whose height is 10 cm, to the nearest tenth.

$$LA = \pi r l$$

$$= \pi (6) (\sqrt{136})$$

$$= 219.8 \text{ cm}^2$$

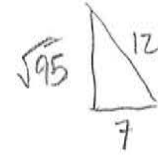


9) Determine the volume of a cone whose radius is 7 in and whose slant height is 12 in, to the nearest tenth.

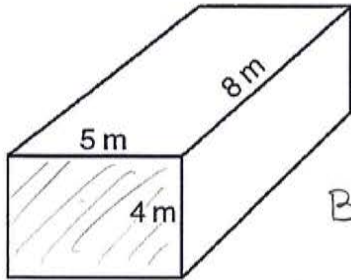
$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (7)^2 \sqrt{95}$$

$$V = 500.1 \text{ in}^3$$



10) Determine the lateral area, surface area, and volume of the figure below:



$$LA = Ph$$

$$= 18 \cdot 8$$

$$LA = 144 \text{ in}^2$$

$$BA = 5 \cdot 4$$

$$= 20$$

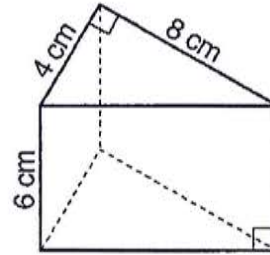
$$SA = 184 \text{ in}^2$$

$$V = Bh$$

$$= 5 \cdot 4 \cdot 8$$

$$V = 160 \text{ m}^3$$

11) Determine the volume of the prism below:



$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3} \left[ \frac{1}{2} \cdot 4 \cdot 8 \right] 6$$

$$V = 32 \text{ cm}^3$$

12) If the volume of a sphere is  $2304\pi \text{ in}^3$ , determine the surface area of the sphere.

$$V = \frac{4}{3} \pi r^3$$

$$1728 = r^3$$

$$SA = 4\pi r^2$$

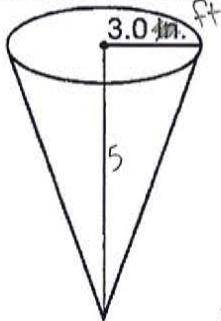
$$576\pi \text{ in}^2$$

$$2304\pi = \frac{4}{3} \pi r^3$$

$$r = 12$$

$$= 4\pi (12)^2$$

13) What is the height of the cone below, to the nearest inch, if the volume is  $47.1 \text{ ft}^3$ ?



$$V = \frac{1}{3} \pi r^2 h$$

$$h =$$

$$47.1 = \frac{1}{3} \pi (3)^2 h$$

$$h = 5 \text{ in}$$

Based on the height you found, determine the surface area of the cone, to the nearest inch

$$LA = \pi r l$$

$$BA = \pi r^2$$

$$= \pi (3) \sqrt{34}$$

$$= \pi (3)^2$$

$$= 54.955427$$

$$= 28.274334$$

$$83.229761 \text{ feet}^2$$

$$SA = 11985 \text{ in}^2$$

**Solids Review # 2**

- 1) A triangular prism has an isosceles right triangular base with a hypotenuse of  $\sqrt{32}$  and a prism height of 15. A square prism has a height of 15, and its volume is equal to that of the triangular prism. What are the dimensions of the square base?

$$V = Bh$$

$$= \frac{1}{2} 4 \cdot 4 \cdot 15$$

$$= 120$$

$$120 = y^2 \cdot 15$$

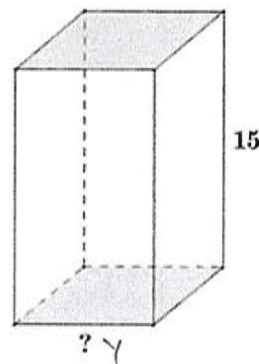
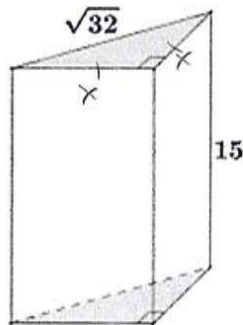
$$8 = y^2$$

$$y = 2\sqrt{2}$$

$$32 = 2x^2$$

$$x^2 = 16$$

$$x = 4$$

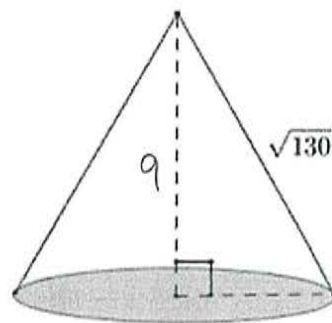


- 2) The right circular cone shown has a base with radius of 7. The slant height of the cone's lateral surface is  $\sqrt{130}$ . Find the volume of the cone.

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (7)^2 9$$

$$V = 147\pi$$



- 3) A circular cylinder has a radius between 5.50 and 6.00 cm and a volume of  $225 \text{ cm}^3$ . Write an inequality that represents the range of possible heights the cylinder can have to meet this criterion to the nearest hundredth of a centimeter.

$$V = \pi r^2 h$$

$$225 = \pi (5.5)^2 h$$

$$h = 2.37$$

$$225 = \pi 6^2 h$$

$$h = 1.99$$

$$1.99 < h < 2.37$$

- 4) A machine part is manufactured from a block of iron with circular cylindrical slots. The block of iron has a width of 14 in., a height of 16 in., and a length of 20 in. The number of cylinders drilled out of the block is determined by the weight of the leftover block, which must be less than 1,000 lb.

- a. If iron has a weight of roughly  $491 \text{ lb/ft}^3$ , how many cylinders with the same height as the block and with radius 2 in. must be drilled out of the block in order for the remaining solid to weigh less than 1,000 lb.?

$$273 = 57x$$

$$V = 14 \cdot 16 \cdot 20$$

$$\frac{4480}{12^3} \approx 491 \approx 1273 \text{ lb}$$

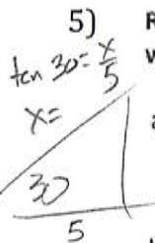
$$\frac{\pi (2)^2 16}{12^3} \cdot 491 = 57$$

$$x = 5 \text{ cyl}$$

- b. If iron ore costs \$115 per ton (1 ton = 2200 lb.) and the price of each part is based solely on its weight of iron, how many parts can be purchased with \$1,500? Explain your answer.

$$1273 - 57(5) = 988$$

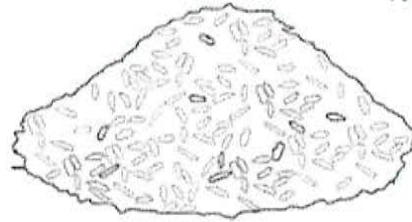
$\frac{7}{6}$   $\frac{4}{3}$   $\frac{5}{3}$



5) Rice falling from an open bag piles up into a figure conical in shape with an approximate radius of 5 cm.

a. If the angle formed by the slant of the pile with the base is roughly  $30^\circ$ , write an expression that represents the volume of rice in the pile.

b. If there are approximately 20 grains of rice in a cubic centimeter, approximately how many grains of rice are in the pile? Round to the nearest whole grain.



$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (5)^2 (2.887)$$

$$= 75.581483$$

$$\cdot 20$$

$$1512 \text{ grains}$$

6) In a solid hemisphere, a cone is removed as shown. Calculate the volume of the resulting solid. In addition to your solution, explain the strategy you used in your solution.

$$V_{\text{SPH}} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \pi (9)^3$$

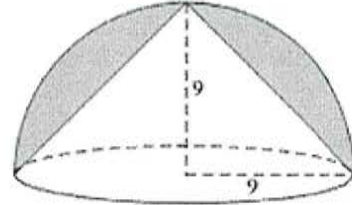
$$= 972 \pi / 2 = 486 \pi$$

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (9)^2 (9)$$


$$= 243 \pi$$

$$\begin{array}{r} 486 \pi \\ - 243 \pi \\ \hline 243 \pi \end{array}$$





7) Describe the shape of the cross-section of each of the following objects.

*Right circular cone:*

a. Cut by a plane through the vertex and perpendicular to the base 

*Square pyramid:*

b. Cut by a plane through the vertex and perpendicular to the base 

c. Cut by a vertical plane that is parallel to an edge of the base but not passing through the vertex 

8) The following is a greenhouse made up of a rectangular prism and a triangular prism. Determine the volume of the greenhouse.

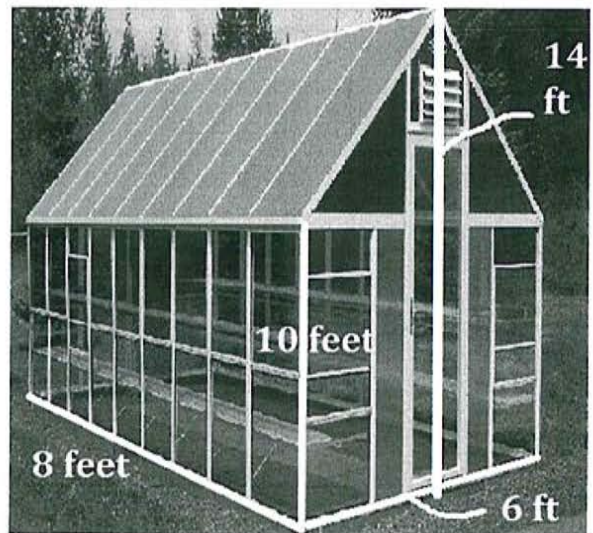
$$V = 8 \cdot 10 \cdot 6$$

$$= 480$$

$$V = \frac{1}{2} (6)(4)(8)$$

$$= 96$$

$$576 \text{ ft}^3$$



If Jessica wants to grow as many tomato plants as possible, and recommendations are that each tomato plant needs 3.2 square feet of room for growth, in addition to the 50 square feet she needs for walking/watering space, how many tomato plants can Jessica put in her greenhouse at max capacity?

$$\begin{array}{r} 576 \\ - 50 \\ \hline 526 \\ \hline 3.2 \end{array} = 164 \text{ plants}$$

9) Two cones are similar. Their radii are 12 in and 18 in, respectively. If the volume of the cone with the 12 in radius is  $720\pi \text{ in}^3$ , determine the volume of the cone with the 18 in radius.

$$\frac{720}{12^3} = \frac{x}{18^3}$$

$$x = 2430 \text{ in}^3$$