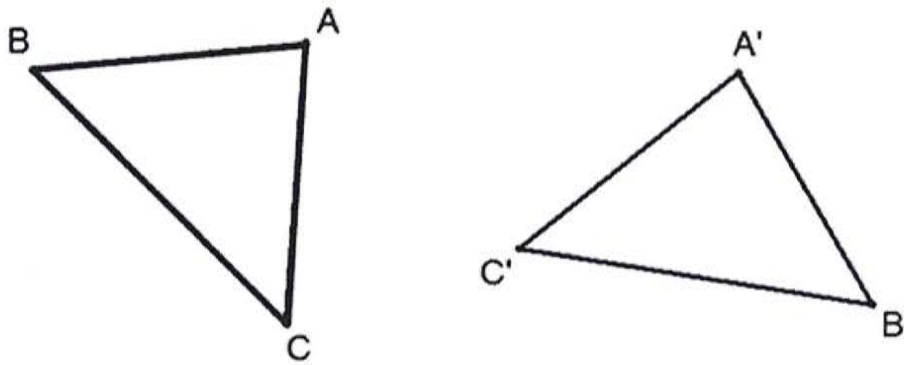


OBJECTIVE: How do we prove that triangles are congruent?

I. Important Statements:

1. Segments can be mapped onto segments **IF AND ONLY IF** they are \cong
2. Angles can be mapped onto angles **IF AND ONLY IF** they are \cong

Definition of Congruent: Two triangles (or figures) are said to be congruent **IF AND ONLY IF** there is a series of rigid motions that maps the figure onto the other.



Given two triangles $\triangle ABC$ and $\triangle A'B'C'$ such that $m\angle A \cong m\angle A'$, $\overline{AC} \cong \overline{A'C'}$, $\overline{AB} \cong \overline{A'B'}$.

Name the missing corresponding sides and angles

Find a composition of rigid motions to map $\triangle ABC$ onto $\triangle A'B'C'$

I. Some new statements that we can use in a proof:

- A. An **EQUILATERAL** triangle has 3 \cong sides
- B. Base angles in an isosceles triangle are \cong
- C. A median of a triangle creates a **MIDPOINT** on one side of the triangle

II. There are 5 ways to prove triangles are congruent

- **SSS – Side Side Side**
- **ASA – Angle Side Angle**
- **AAS – Angle Angle Side**
- **SAS – Side Angle Side**
- **HL – Hypotenuse Leg**

III. How to Prove Triangles are Congruent

Step 1: Write down the given information

Step 2: Use the given information to determine what you know about the picture

- Mark off congruent pieces on the picture after stating them in the proof

Step 3: The only congruency not from the given will be

- **REFLEXIVE**
- **VERTICAL ANGLES**

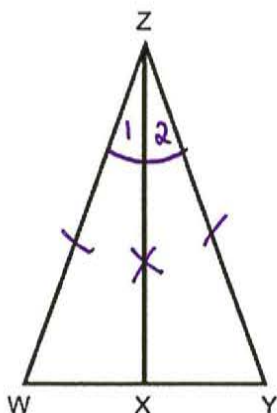
Step 4: Your last statement is **ALWAYS** the **PROVE**

Step 5: Your last reason is always one of the 5 ways to prove triangles are congruent

1. Given: Isosceles $\triangle ZYW$

\overline{XZ} bisects $\angle WZY$

Prove $\triangle WZX \cong \triangle YZX$



S

① Iso $\triangle ZYW$
 \overline{XZ} bisects $\angle WZY$

② $\overline{WZ} \cong \overline{YZ}$

③ $\angle 1 \cong \angle 2$

④ $\overline{XZ} \cong \overline{XZ}$

⑤ $\triangle WZX \cong \triangle YZX$

R

① Given

② An Iso \triangle has 2 \cong sides

③ An \angle bisector creates
2 \cong \angle s

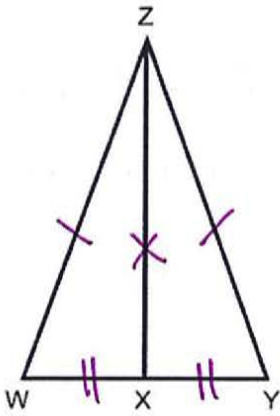
④ Reflexive

⑤ SAS

2. Given: Isosceles $\triangle ZYW$

\overline{XZ} bisects \overline{WY}

Prove $\triangle WZX \cong \triangle YZX$



S

① Iso $\triangle ZYW$
 \overline{XZ} bisects \overline{WY}

② $\overline{WZ} \cong \overline{YZ}$

③ $\overline{XZ} \cong \overline{XZ}$

④ X is a mdpt of \overline{WY}

⑤ $\overline{WX} \cong \overline{YX}$

⑥ $\triangle WZX \cong \triangle YZX$

R

① Given

② An iso \triangle has 2 \cong sides

③ Reflexive

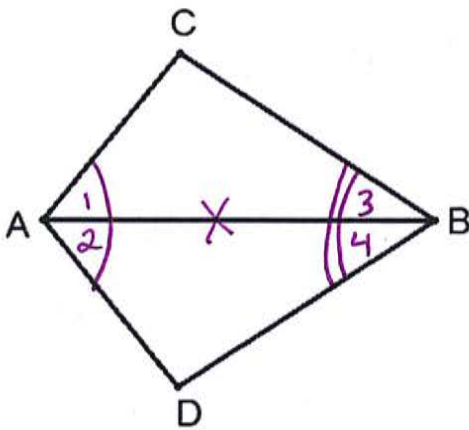
④ Seg bisector creates a mdpt

⑤ Mdpt creates 2 \cong segments

⑥ SSS

3. Given \overline{AB} bisects both $\angle CAD$ and $\angle CBD$

Prove $\triangle ABD \cong \triangle ABC$



S

① \overline{AB} bisects both
 $\angle CBD$ & $\angle CAD$

② $\angle 1 \cong \angle 2$
 $\angle 3 \cong \angle 4$

③ $\overline{AC} \cong \overline{BC}$

④ $\triangle ABD \cong \triangle ABC$

R

① Given

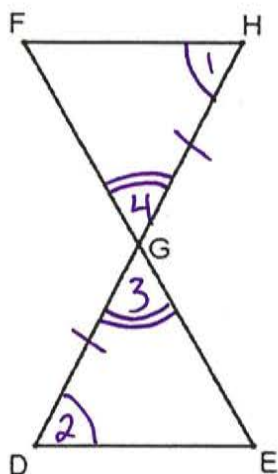
② An \angle bisector creates
2 \cong \angle s

③ Reflexive

④ ASA

4. Given $\overline{FH} \parallel \overline{DE}$, G is the midpoint of \overline{DH}

Prove $\triangle FHG \cong \triangle EDG$

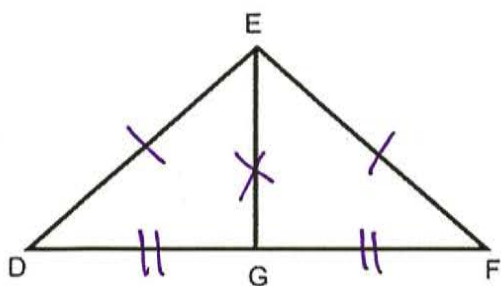


- S
- ① $\overline{FH} \parallel \overline{DE}$
G is mdpt of \overline{DH}
 - ② $\angle 1 \cong \angle 2$
 - ③ $\overline{DG} \cong \overline{HG}$
 - ④ $\angle 3 \cong \angle 4$
 - ⑤ $\triangle FHG \cong \triangle EDG$

- R
- ① Given
 - ② IF 2 \parallel lines are cut by a transversal, alt int are \cong
 - ③ A mdpt creates 2 \cong seg
 - ④ Vertical \angle s are \cong
 - ⑤ ASA

5. Given \overline{EG} is a median of $\triangle DEF$, $DE = FE$

Prove $\triangle DEG \cong \triangle FEG$

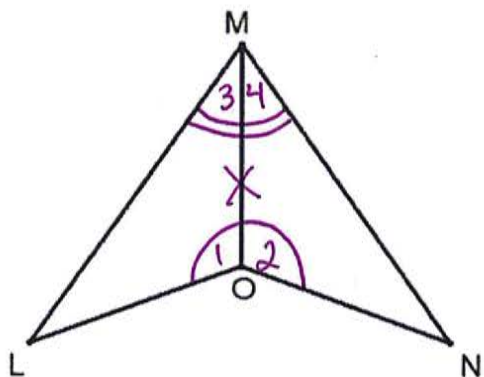


- S
- ① \overline{EG} is median of $\triangle DEF$
 $DE = FE$
 - ② G is the mdpt of \overline{DF}
 - ③ $DG = FG$
 - ④ $EG = EG$
 - ⑤ $\triangle DEG \cong \triangle FEG$

- R
- ① Given
 - ② A median creates a mdpt.
 - ③ A mdpt creates 2 = segments
 - ④ Reflexive
 - ⑤ SSS

6. Given \overline{OM} bisects $\angle LMN$, $\angle LOM \cong \angle NOM$

Prove $\triangle LMO \cong \triangle NMO$



S
① \overline{OM} bisects $\angle LMN$

$$\angle LOM \cong \angle NOM$$

$$\angle 1 \cong \angle 2$$

② $\angle 3 \cong \angle 4$

③ $\overline{MO} \cong \overline{MO}$

④ $\triangle LMO \cong \triangle NMO$

R
① Given

② An \angle bisector creates
 $2 \cong \angle s$

③ Reflexive

④ ASA

7. If \overline{AD} is the perpendicular bisector of \overline{BC} ,

then prove $\triangle ABD \cong \triangle ACD$



S
① \overline{AD} is \perp bisector of \overline{BC}

② $\angle 1$ & $\angle 2$ are rt $\angle s$

③ $\angle 1 \cong \angle 2$

④ D is the mdpt of \overline{BC}

⑤ $\overline{BD} \cong \overline{CD}$

⑥ $\overline{AD} \cong \overline{AD}$

⑦ $\triangle ABD \cong \triangle ACD$

R
① Given

② \perp lines form rt $\angle s$

③ All rt $\angle s$ are \cong

④ A segment bisector
creates a mdpt

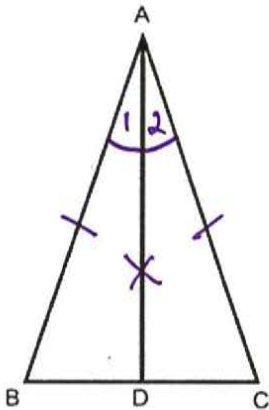
⑤ A mdpt creates $2 \cong$ seg

⑥ Reflexive

⑦ SAS

8. Given $AB = AC$, \overline{DA} bisects $\angle BAC$

Prove $\triangle ABD \cong \triangle ACD$



① $AB = AC$
 \overline{DA} bisects $\angle BAC$

② $\angle 1 \cong \angle 2$

③ $\overline{AD} \cong \overline{AD}$

④ $\triangle ABD \cong \triangle ACD$

S

R

① Given

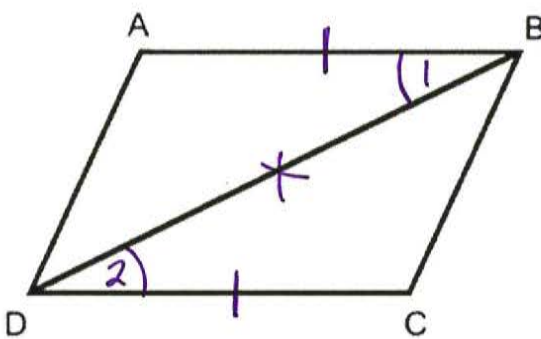
② An \angle bisector creates
 $2 \cong \angle s$

③ Reflexive

④ SAS

9. Given: $\overline{AB} \parallel \overline{CD}$ and $AB = CD$

Prove $\triangle ABD \cong \triangle CBD$



① $\overline{AB} \parallel \overline{CD}$
 $AB = CD$

② $\angle 1 \cong \angle 2$

③ $\overline{DB} \cong \overline{BD}$

④ $\triangle ABD \cong \triangle CBD$

S

R

① Given

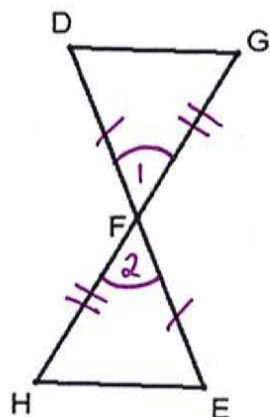
② If 2 \parallel lines are cut
by a transversal, alt int $\angle s \cong$

③ Reflexive

④ SAS

10. Given \overline{DG} & \overline{EH} bisect each other at F

Prove: $\triangle DGF \cong \triangle EHF$

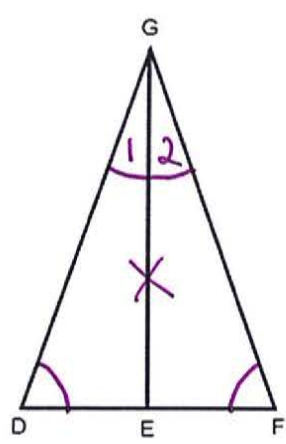


$$\triangle DGF \cong \triangle EHF$$

- S
- ① \overline{DG} & \overline{EH} bisect at F
 - ② F is mdpt of \overline{DE} & \overline{GH}
 - ③ $\overline{DF} \cong \overline{EF}$
 $\overline{GF} \cong \overline{HF}$
 - ④ $\angle 1 \cong \angle 2$
 - ⑤ $\triangle DGF \cong \triangle EHF$

- R
- ① Given
 - ② Segment bisector creates a mdpt.
 - ③ A mdpt creates 2 \cong segments
 - ④ Vertical \angle s are \cong
 - ⑤ SAS

11. Given: $m\angle D = m\angle F$
 \overline{EH} bisects $\angle DGF$
Prove: $\triangle DGE \cong \triangle FGE$

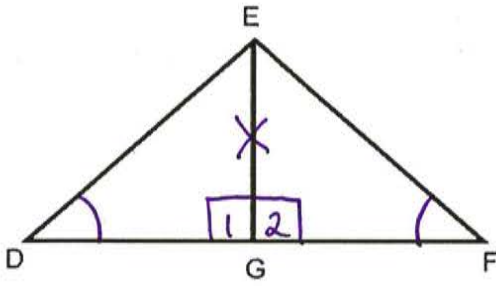


- S
- ① $m\angle D = m\angle F$
 \overline{EH} bisects $\angle DGF$
 - ② $\angle 1 \cong \angle 2$
 - ③ $\overline{GE} \cong \overline{GE}$
 - ④ $\triangle DGE \cong \triangle FGE$

- R
- ① Given
 - ② An \angle bisector creates 2 \cong \angle s
 - ③ Reflexive
 - ④ AAS

12. Given $\angle D \cong \angle F, \overline{EG} \perp \overline{DF}$

Prove $\triangle DEG \cong \triangle FEG$



S

① $\angle D \cong \angle F$

$\overline{EG} \perp \overline{DF}$

② $\angle 1$ & $\angle 2$ are rt \angle s

③ $\angle 1 \cong \angle 2$

④ $\overline{EG} \cong \overline{EG}$

⑤ $\triangle DEG \cong \triangle FEG$

R

① Given

② \perp lines form rt \angle s

③ All rt \angle s are \cong

④ Reflexive

⑤ AAS